



# ARROYO CENTER

CHILD POLICY  
CIVIL JUSTICE  
EDUCATION  
ENERGY AND ENVIRONMENT  
HEALTH AND HEALTH CARE  
INTERNATIONAL AFFAIRS  
NATIONAL SECURITY  
POPULATION AND AGING  
PUBLIC SAFETY  
SCIENCE AND TECHNOLOGY  
SUBSTANCE ABUSE  
TERRORISM AND  
HOMELAND SECURITY  
TRANSPORTATION AND  
INFRASTRUCTURE

This PDF document was made available from [www.rand.org](http://www.rand.org) as a public service of the RAND Corporation.

[Jump down to document ▼](#)

The RAND Corporation is a nonprofit research organization providing objective analysis and effective solutions that address the challenges facing the public and private sectors around the world.

## Support RAND

[Purchase this document](#)

[Browse Books & Publications](#)

[Make a charitable contribution](#)

## For More Information

Visit RAND at [www.rand.org](http://www.rand.org)

Explore [RAND Arroyo Center](#)

View [document details](#)

## Limited Electronic Distribution Rights

This document and trademark(s) contained herein are protected by law as indicated in a notice appearing later in this work. This electronic representation of RAND intellectual property is provided for non-commercial use only. Permission is required from RAND to reproduce, or reuse in another form, any of our research documents for commercial use.

This product is part of the RAND Corporation technical report series. Reports may include research findings on a specific topic that is limited in scope; present discussions of the methodology employed in research; provide literature reviews, survey instruments, modeling exercises, guidelines for practitioners and research professionals, and supporting documentation; or deliver preliminary findings. All RAND reports undergo rigorous peer review to ensure that they meet high standards for research quality and objectivity.

# TECHNICAL REPORT

---

## The Weapons Mix Problem

A Math Model to Quantify the  
Effects of Internetting of Fires to  
the Future Force

Christopher G. Pernin, Louis R. Moore

Prepared for the United States Army

Approved for public release; distribution unlimited



RAND ARROYO CENTER

Report Documentation Page				Form Approved OMB No. 0704-0188	
Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.					
1. REPORT DATE <b>2005</b>		2. REPORT TYPE <b>N/A</b>		3. DATES COVERED <b>-</b>	
4. TITLE AND SUBTITLE <b>The Weapons Mix Problem: A Math Model to Quantify the Effects of Internetting of Fires to the Objective Future Force</b>				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S) <b>Christopher G. Pernin, Louis R. Moore</b>				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>Rand Corporation</b>				8. PERFORMING ORGANIZATION REPORT NUMBER <b>TR-170-A</b>	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT <b>Approved for public release, distribution unlimited</b>					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT <b>For the Armys future force, what is the appropriate mix of weapons to provide a given outcome, and how might these weapons be employed? This research offers some initial observations into the internetting of fires process, the ability to engage a particular target using any number of potential firers who are able to engage due to being on the network which provides targeting information, along with a foundation for understanding its relationship to combat outcome.</b>					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT <b>UU</b>	18. NUMBER OF PAGES <b>46</b>	19a. NAME OF RESPONSIBLE PERSON
a. REPORT <b>unclassified</b>	b. ABSTRACT <b>unclassified</b>	c. THIS PAGE <b>unclassified</b>			

The research described in this report was sponsored by the United States Army under Contract No. DASW01-01-C-0003.

ISBN: 0-8330-3781-1

The RAND Corporation is a nonprofit research organization providing objective analysis and effective solutions that address the challenges facing the public and private sectors around the world. RAND's publications do not necessarily reflect the opinions of its research clients and sponsors.

**RAND®** is a registered trademark.

© Copyright 2005 RAND Corporation

All rights reserved. No part of this book may be reproduced in any form by any electronic or mechanical means (including photocopying, recording, or information storage and retrieval) without permission in writing from RAND.

Published 2005 by the RAND Corporation  
1776 Main Street, P.O. Box 2138, Santa Monica, CA 90407-2138  
1200 South Hayes Street, Arlington, VA 22202-5050  
201 North Craig Street, Suite 202, Pittsburgh, PA 15213-1516  
RAND URL: <http://www.rand.org/>  
To order RAND documents or to obtain additional information, contact  
Distribution Services: Telephone: (310) 451-7002;  
Fax: (310) 451-6915; Email: [order@rand.org](mailto:order@rand.org)

## Preface

---

Network Centric Warfare, the new paradigm of future warfighting, will produce increased amounts of information, and new tools will be needed to better utilize that information. With the increase in the flows of information, decisionmaking tools and processes from the strategic to the tactical level will allow force elements to be used more effectively during a campaign. Understanding the appropriate mixes of effects-generating capabilities necessary to provide a given measure of outcome—and how these capabilities might be employed in the network-centric future—is the driver for the work on internetting of fires (IOF). Simply put, IOF is “the ability to engage a particular target using any number of potential firers who are able to engage due to being on the network which provides targeting information.”[1]

The goal of this report is to describe a method for answering the question, How might internetted weapons be best employed? Additionally, this study provided a better understanding of the IOF process and a means to quantify its relationship to combat outcome.

An important aspect of implementing the IOF concept will be to discover how best to allocate fire missions from a collection of shooters on a network. This report describes the design and use of an analytical tool to assist in determining the allocation of weapons to targets. Proof-of-principle examples that demonstrate the model’s utility are given, along with observations and a discussion on the way ahead for this methodology.

The tool was designed to be simple, unencumbered, and transparent, enabling the customer to use it quickly to develop insights into weapon allocation and other aspects of future battle command systems. This work should be of interest to those involved in C4ISR design, development, and system acquisition planning for the Army’s Future Force.

This research was sponsored by the Director of TRADOC Analysis Center (TRAC) and was conducted in RAND Arroyo Center’s Force Development and Technology Program. RAND Arroyo Center, part of the RAND Corporation, is a federally funded research and development center sponsored by the United States Army.

For more information on RAND Arroyo Center, contact the Director of Operations (telephone 310-393-0411, extension 6419; FAX 310-451-6952; e-mail [Marcy\\_Agmon@rand.org](mailto:Marcy_Agmon@rand.org)), or visit Arroyo's web site at <http://www.rand.org/ard/>.

# Contents

---

Preface .....	iii
Figures .....	vii
Tables .....	ix
Summary .....	xi
Acknowledgments .....	xv
Acronyms .....	xvii
 <b>CHAPTER ONE</b>	
<b>Introduction</b> .....	1
Networked Fires .....	2
Structure of the Report .....	4
 <b>CHAPTER TWO</b>	
<b>Description of Methodology</b> .....	5
Problem Formulation .....	6
Data Input .....	6
Constraints .....	8
Origin of the Input Data .....	8
Calculating Expected Kills .....	9
Limitations of the Model .....	10
 <b>CHAPTER THREE</b>	
<b>Example Analysis</b> .....	11
 <b>CHAPTER FOUR</b>	
<b>Insights and Future</b> .....	17
Future Improvements .....	17
 <b>APPENDIX</b>	
A. Mathematical Formulation of the Problem .....	19
B. Run-Time Analysis .....	25
References .....	27





## Figures

---

S.1.	The IOF Allocator as Part of a Suite of Analytical Tools .....	xii
1.	Internetting of Fires Is the Dynamic Pooling of Resources Enabled by C4ISR .....	3
2.	The IOF Allocator as Part of the WMT Suite of Analytical Tools .....	5
3.	Allocation of Fires .....	11
4.	Allocation of Fires If the Penalty for Collateral Targets Is Increased .....	13
5.	Allocation of Fires If the Overkill Factor Is Decreased.....	14
6.	Allocation of Fires If the Value of the Close Targets Is Increased.....	15
B.1.	Flow Chart of Model Process .....	25



## Tables

---

1. Sources for Some of the Data in the Study.....	9
B.1. Run-time Analysis of the Different Steps Involved in the Methodology .....	26



## Summary

---

The maneuver units of action (UA) in emerging U.S. Army Future Force<sup>1</sup> operational concepts will need to “see first, understand first, act first, and finish decisively” [5] in order to achieve battlefield success. This mandate implies the utilization of information on the battlefield quickly and effectively across platforms, echelons, and services. The future as exemplified by Network Centric Operations is that there will be an increase in the amount of information produced and made available across the military [2, 3], which will need to be understood and used to make the best decisions on force employment. Without the ability to process, understand, and utilize the increased amount of information generated on the battlefield, the expectations of Network Centric Operations will not be met.

Internetting of fires is “the ability to engage a particular target using any number of potential firers who are able to engage due to being on the network, which provides targeting information.” [1] Implied by this definition is the management of knowledge of the battlefield and the acceptance of coherent and consistent decisionmaking criteria to effectively execute an engagement. These two ideas, a shared awareness of the battlefield and a consistent decisionmaking capability, are necessary conditions for self-synchronization among forces. Internetting of fires conforms to the tenets of Network Centric Warfare, and it provides the basis for self-synchronization among firers.

To date, it has been difficult to quantitatively assess the effects of decisionmaking processes and information flows in the targeting of opponents, as well as their effects on force structure and force effectiveness. With the increase in the flows of information, decisionmaking tools and methods from the strategic to the tactical level will be needed to make better use of the information and to utilize force elements more effectively during a campaign. Understanding the appropriate mixes of effects-generating capabilities necessary to provide a given measure of outcome—and how these capabilities might be employed in the network-centric future—is the driver for the work on internetting of fires.

The goal of this report was to describe a method for answering the question, How might internetted weapons be best employed? Additionally, this study provided a better understanding of the IOF process and a means to quantify its relationship to combat outcome.

An important part of implementing the IOF concept is to devise a consistent and optimized decisionmaking process to match weapons to targets on the battlefield. This report describes the design and use of an analytical decisionmaking tool to assist in that allocation. This tool and the model described could be used in constructive simulations and may provide insights into the use of such tools as real-time decision aids during combat. Proof-of-

---

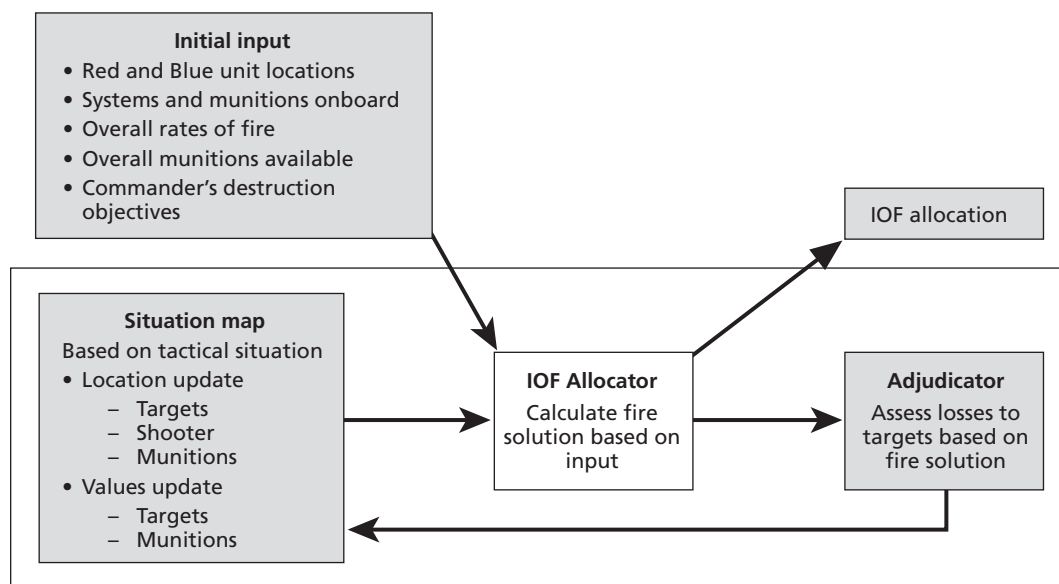
<sup>1</sup> The term “Future Force” has replaced the prior “Objective Force.”

principle examples that demonstrate the model's utility are given, along with observations and a discussion of the way ahead for this methodology.

Simply put, the tool takes what is known about a decisionmaker's current stock of weapons, along with what potential targets exist that are to be fired upon, and produces the best set of fire missions to be tasked. Because the tool is making a decision based on a snapshot of the battlefield, it is formulated to maximize the *expected* value of targets killed subject to several constraints. The relative values of targets are obtained from an assessment of target significance (ATS), which is a commander's dynamic measure of a particular target's importance with respect to the accomplishment of his plan. The decision variables in the program are the number of missions executed by specific shooters and munitions against targets. Constraints are imposed for shooter firing rate, munitions range and availability, budget (for example, cost or weight), killing limits, and other rules of engagement (ROE). The mathematical formulation at the heart of the IOF Allocator is provided in detail in Appendix A to this report.

An allocation of fires is developed through an iterative process (see Figure S.1). The initial input data include the allowed munitions types for each type of shooter, their ranges, and their rate of fire. In addition, the percentage kill limits for each target type, the future cost of each munitions type (on the same scale as the value of a target), and the overall budget is input. A key driver for the process is the performance of each munitions type against each target type by range. The IOF Allocator works from a situation map with shooter, target, and weapon locations as well as target values and kill limits. The Allocator then computes optimal assignments of shooters to targets based upon the input, and the Adjudicator assesses losses to the targets based on these assignments. Those results, modified by user judgment, then feed back into the situation map. The process iterates until an acceptable allocation of fires is obtained, which meets the decisionmaker's expectations for target attrition.

**Figure S.1**  
**The IOF Allocator as Part of a Suite of Analytical Tools**



The tool was used by TRAC in weapons mix analyses for the Future Force. To date, the tool has allowed the formulation of follow-on questions that may be answered by a thorough analysis with the described method. The questions may help to identify the sorts of conclusions that could be expected from a more complete analysis with the methodology described in this study. The questions can be summarized as follows:

- Is the selection of possible targets driven by available weapons?
- What is the effect of weapon accuracy on the choice of targets?
- What is the role in future forces for area munitions, and what is the appropriate mix of area versus point munitions?
- How do the contributions of various weapon systems change with changing enemy force composition and disposition?
- How dependent on range is the utilization of specific weapon systems?

This report has produced a tool for answering the two goals of this study: What is the appropriate mix of weapons to provide a given outcome, and how might these weapons be employed in the future? In doing so, we have produced some initial observations into the internetting of fires process and a foundation for understanding its relationship to combat outcome.





## Acknowledgments

---

The authors would like to express our thanks to Mr. Mike Bauman (Director of TRADOC Analysis Center) for his input and support of this project. Our points of contact at TRAC, MAJ Robert Bradford and Mr. Chris Morey, were instrumental in providing the information necessary to run the model, and they also gave us invaluable insights into the Army.

We would also like to thank Richard Stanton (RAND) and MAJ Robert Kewley for their reviews of and helpful comments on the report. Others at RAND who provided help in the preparation and review of the document include Ken Horn, Leland Joe, Kristin Leuschner, and Patrice Lester.

While many people were instrumental in bringing this publication to fruition, the analysis and conclusions are the sole responsibility of the authors.



## Acronyms

---

AMSAA	Army Materiel Systems Analysis Activity
ATO	Air Tasking Order
ATS	Assessment of Target Significance
C4ISR	Command, Control, Communications, Computers, Intelligence, Surveillance, and Reconnaissance
DCSINT	Deputy Chief of Staff for Intelligence
FCS	Future Combat Systems
GAMS	General Algebraic Modeling System
IOF	Internetting of Fires
NCW	Network Centric Warfare
NLOS	Non-Line-of-Sight
ROE	Rules of Engagement
SPH	Self Propelled Howitzer
TLE	Target Location Error
TRAC	TRADOC Analysis Center
TRADOC	Training and Doctrine Command
UA	Unit of Action
USAFAS	U.S. Army Field Artillery School
VIC	Vector in Commander
WMT	Weapons Mix Tool



## Introduction

---

The maneuver units of action (UA) in emerging U.S. Army Future Force<sup>1</sup> operational concepts will need to “see first, understand first, act first, and finish decisively” [5] in order to achieve battlefield success. This mandate implies the utilization of information on the battlefield quickly and effectively across platforms, echelons, and services. The future, as exemplified by Network Centric Operations, is that there will be an increase in the amount of information produced and made available across the military [2, 3], which will need to be understood and used to make the best decisions on force employment. Without the ability to process, understand, and utilize the increased amount of information generated on the battlefield, the expectations of Network Centric Operations will not be met.

The input to a headquarters can arrive in different forms. The scope of different inputs can be seen as an extension of information sciences and knowledge management. [17] The input to the headquarters is in the form of data, information, knowledge or wisdom, and the effect that the headquarters has is to add value to the inputs. By moving the input along the path toward wisdom, the headquarters has (hopefully) informed the decision that has to be made.

The question arises, What different types of information exist as inputs into the headquarters? The scale has to do with information *complexity*, which has changed considerably over time and continues to develop. The complexity of information has many factors that drive it, including the number and types of interactions of the information elements, in addition to the rate at which those elements change over time.

The inputs can suffer from differing conceptual frameworks that will drive the interpretation. If two competing frameworks are present, the inputs can take on a complexity that was not present in the presence of only one framework. The military drive toward Effects Based Operations (EBO) highlights the complexity in decisionmaking that can arise when coping with the increased information generated from Network Centric Operations. As the decisionmaking criteria are further developed to include not only military but political and economic tools, the supporting infrastructure to deal with these situations must also adapt.

This report addresses the decision surrounding the use of weapons.<sup>2</sup> The concept of “networked fires” will afford an increase in the available information for making decisions on firing and thus will increase the complexity of the decisionmaking. The complexity comes from an increase in the number of potential targets any one weapon can hit, and an increase

---

<sup>1</sup> The term “Future Force” has replaced the prior “Objective Force.”

<sup>2</sup> “Weapon” and “munition” are used interchangeably in this report, as the majority of the weapons we considered had only one possible munition associated with it.

in the number of potential weapons a decisionmaker can employ. The concept is also referred to as the internetting of fires. Internetting of fires is “the ability to engage a particular target using any number of potential firers who are able to engage due to being on the network, which provides targeting information.” [1] Implied by this definition is the management of knowledge of the battlefield and the acceptance of coherent and consistent decisionmaking criteria to effectively execute an engagement. These two ideas, a shared awareness of the battlefield and a consistent decisionmaking capability, are necessary conditions for self-synchronization among forces.

## Networked Fires

The U.S. Army has adopted a working definition of its vision for the way the Future Force will employ its effects-generating capabilities (weapons). Their vision of networked fires is based on weapons, sensors, and command and control systems rapidly and decisively responding to a commander’s needs. The vision is as follows [9]:

Networked fires includes the triad of relevant sensors, NLOS [non-line-of-sight] fires capabilities (to include access to joint), and battle command that enables dynamic, on-demand, NLOS fires to achieve the UA and subordinate commander’s tactical objectives. Networked fires applies effects-based solutions to achieve the commander’s objectives through the integrated application of lethal and non-lethal munitions and other effects.

Networked fires, therefore, could be an essential component of the Battle Command System.<sup>3</sup> For Networked Fires to come to fruition, the military will need to fully leverage relevant Army, Joint, national, and multinational sensors, weapons, and command structures. The concept entails the integration of platforms such as sensors and weapons with theater-level planning and decisionmaking that will allow the employment of firing capabilities from all echelons. Contrary to how weapons are employed today, weapon use will not be controlled by who owns what (e.g., dedicated fires), but rather by the use of the best weapon across all potential employments.

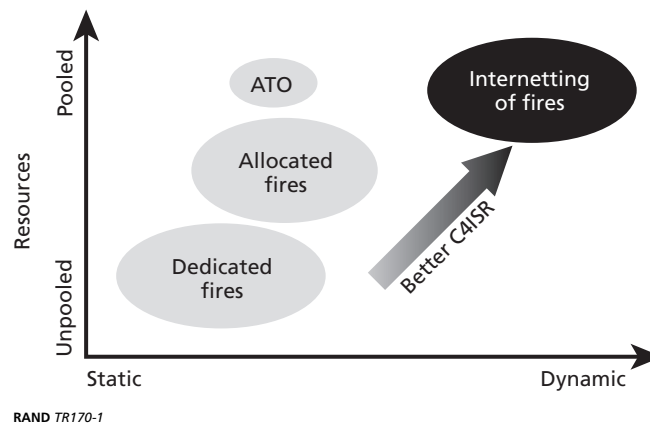
“Internetting of fires” (IOF) may enable the Army’s vision. One definition of the concept is “the ability to engage a particular target using any number of potential firers who are able to engage due to being on the network which provides targeting information.” [1] IOF is a Network Centric Warfare (NCW) concept [2, 3] for dynamically synchronizing pooled effects-generation systems (Figure 1). Key to the concept is the advanced Future Force C4ISR architecture, which will provide the necessary information and a consistent and coherent decisionmaking capability. [4]

As opposed to IOF, dedicated fires entail the static allocation of unpooled resources. In dedicated fires, weapons are allocated early in an engagement and commanders have little recourse to change this allocation throughout the engagement. The shooters are part of a collection determined well in advance, which may not be mission specific.

---

<sup>3</sup> The Army Battle Command System (ABCS) is a system of systems designed to provide essential information to the commander.

**Figure 1**  
**Internetting of Fires Is the Dynamic Pooling of Resources Enabled by C4ISR**



An example of allocation from a pooled resource may be the Air Tasking Order (ATO) used by joint forces to allocate fires during a campaign:

The Air Tasking Order (ATO) is a representation of a Joint Air Operations Plan. The course of action for air operations is implemented through the Joint Air Operations Plan. The joint air operation plan documents the plan for integrating and coordinating joint air operations. The plan encompasses operations of capabilities/forces from joint force components. [10]

In an ATO, resources from multiple joint force components are pooled to deliver effects across the battlefield. The allocation, however, is historically static: resources are allocated for a mission planned out many hours in advance, and this is not easily updated over time. As an example, during Operation Desert Storm in the Persian Gulf, the ATO took two days to rewrite.<sup>4</sup> The allocation of aircraft sorties through the Tactical Control System is routinely more dynamic. However, the designation of weapons to specific targets is not. The Air Force ATO is often passed manually to Navy aircraft carriers. [11] The Air Force has been interested in enhancing its dynamic command and control and battle management (DC2BM) of air assets in operations against time critical targets (TCT) in conflicts [12] to reduce the cycle time to detect, classify, recognize, and defeat elusive targets.

As the allocation of resources moves from static to dynamic, new pressures are put on C4ISR systems to allocate effects-generating capabilities to where they are needed on the battlefield, when they are needed. The dynamic allocation of fires during conflict entails a new concept of operations in which plans are made, remade, and updated while missions are being carried out. For this to occur, new tools will be necessary to help decisionmakers sort and use the large amount of available information. Both in simulations and as real-time decision aids, representing these decisions on the application of fires will be increasingly important.

The concept of networked fires has gained acceptance as a pivotal enabler of the Future Force concept. However, implementing networked fires has posed a challenge to the Army. The TRADOC Analysis Center (TRAC) C4ISR Modeling and Analysis Work Group

<sup>4</sup> It must be noted that sometimes sorties were initially unassigned and then tasked on-the-fly during combat, which reduced the time between the tasking and the mission engagement.



found that organization, doctrine, and tactics for networked fires are not well defined. Defining the concepts for access, grouping, and control of both organic and nonorganic sensors and weapons are major challenges to the Army. [13] In the spring of 2002, TRAC began a weapons and munitions mix analysis to investigate what combinations of weapons and munitions enable the Future Force to execute its operational networked fires concept. That study was to consider a broad range of weapons mixes across a spectrum of scenarios and operational conditions.

A key problem with implementing the concept of networked fires in analysis tools is the inability to dynamically allocate fires against multiple targets or groups of targets. Usually such allocations are static and represent doctrinal concepts based on dedicated fires. Other times they are derived from rules of thumb developed by subject matter experts. These rules tend to be too myopic to represent Network Centric Warfare accurately.

The allocation of fires is not a new concept; however, the Network Centric Warfare vision of allocating fires introduces a degree of complexity not previously encountered. [16] The complexity can arise from four components: an increase in the amount of information elements becoming available to forces, the degree of interaction of these elements, the dynamics of how fast the information elements are changing, and the predictability of those changes. The networked vision of forces implies an increase in the first two factors. That is, a connection to information networks will change not only the amount of information available, but also the types of information possible to inform decisionmaking. Similarly, an increased focus on the secondary and tertiary affects of targeting operations will necessarily increase the interdependency of decisions that are made and require more timely updating of situations to ensure that operational and strategic expectations are being met. The concept of networked fires will necessitate a step change in how the military utilizes the information it gathers.

Since a networked force will utilize assets differently than ever before, it is important to understand what effects this networking will have on force structure and force effectiveness. The goal of this report was to describe a method for answering this question: How might internetted weapons be best employed? Additionally, this study provided a better understanding of the IOF process and a means to quantify its relationship to combat outcome.<sup>5</sup>

## Structure of the Report

This report describes a mathematical tool for use by a decisionmaker to help determine how best to employ weapons against enemy targets. This tool is known here as the IOF Allocator. Chapter Two formulates the problem and identifies what data are necessary to run the tool and what outputs the tool provides. Chapter Three provides an example analysis using the tool from a simple scenario. Chapter Four provides some observations from the example analysis, along with additional questions that may be answered from a more complete and rigorous analysis.

---

<sup>5</sup> There have subsequently been other attempts at understanding these concepts. [14, 15]

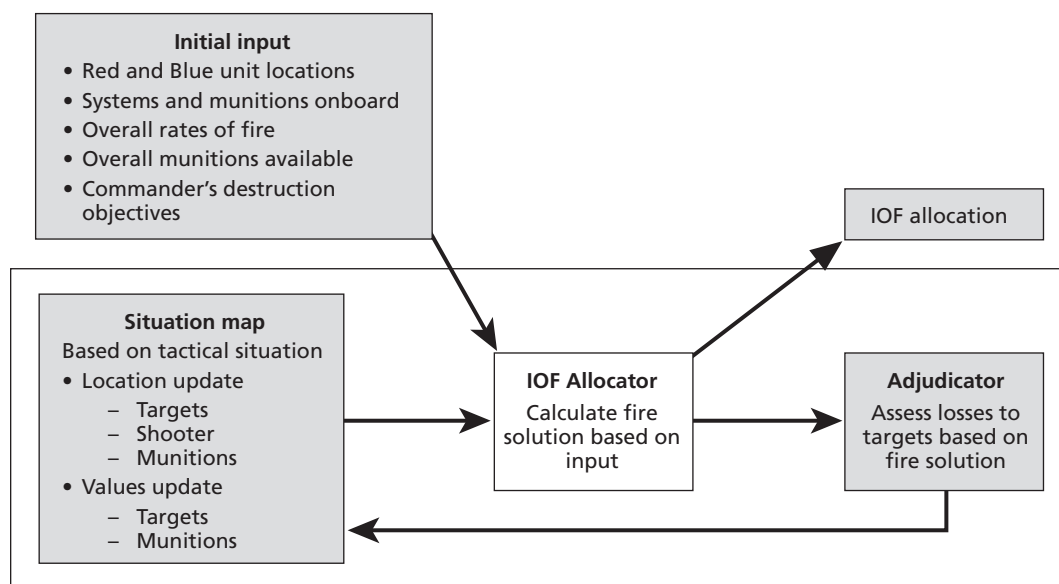
## Description of Methodology

---

A key problem in implementing the concept of internetting of fires is how decisionmakers will assign the most appropriate weapon against a given target. The solution we propose is to develop a common method that decisionmakers can utilize and apply to their perception of the battlespace to determine the allocation of weapons across a battle or campaign. We formalized this decisionmaking method in an analytical tool for performing that allocation. This tool, the IOF Allocator, will be used to help Future Force analysts screen for good mixes of weapons, munitions, and sensors.

During the conduct of this research, we focused on the development of a tool (the IOF Allocator) that assigns shooters to targets based on available information. The tool would be one of several necessary to facilitate the decisionmaking process as it applies to networked fires. Thus, we envision the IOF Allocator as a member of a suite of models called the Weapons Mix Tool (WMT). The WMT develops a situation map, allocates fires, and adjudicates that allocation. The process can iterate for each step until the desired results are obtained (Figure 2). From this suite of models, a decisionmaker can determine not only the best mix of weapons to be used over a given set of scenarios, but also how best to assign those weapons to targets on the battlefield.

**Figure 2**  
The IOF Allocator as Part of the WMT Suite of Analytical Tools



The initial input data include the allowed munitions types for each type of shooter, their ranges, and their rates of fire. In addition, the kill limits for each target type, the future cost of each munition type, and the overall budget are supplied by the user. A key input is the performance of each munition type against each target type by range. The IOF Allocator works from a situation map with shooter, target, and munitions locations as well as target values and kill limits. The Allocator then computes an optimal allocation of fire missions based upon the input, and the Adjudicator assesses losses to the targets based on this allocation. Those results then feed back into the situation map. The process iterates until the allocation of fires provides the decisionmaker with the desired level of attrition to the targets.

The IOF Allocator works above the tactical level of combat, an area where perhaps military judgment is most appropriate. However, the tool may be useful for complicated allocation problems and for the combat adjudication of the firing solution.

## Problem Formulation

This section will describe how we formulated the problem of determining an optimized set of fire missions.

The solution was implemented as a mixed-integer math program solved currently in GAMS [6] using the CPLEX solver.<sup>1</sup> The data are generated from a combination of Microsoft Access and Excel, and they are output as flat text files that are brought into a variety of programs for analysis. The details and full description of the mathematical model at the heart of the IOF Allocator and its implementation are included in Appendixes A and B. Here we will cover the details of the data and constraints in the model.

The allocation tool maximizes the expected value of targets killed resulting from a set of missions.<sup>2</sup> Target values are obtained from an assessment of target significance (ATS), which is a commander's dynamic measure of the importance associated with engaging a target to enable accomplishment of his plan. As this is a relative scale, the appropriate value for each target will come from a judgment among all targets as to their relative values. The decision variables in the math program are the number of fire missions executed by specific shooters and munitions against targets. Constraints are imposed for shooter firing rate, munitions range and availability, budget, killing limits, and rules of engagement (ROE). The math program outputs incremental or marginal values for various alternatives while specifically solving for those alternatives.

## Data Input

The data used in the math program are provided as text files collected from a variety of sources (discussed later in this chapter). The geographical positions of shooters, munitions, and targets use an identical location methodology based on the military grid reference system. The grid can be portrayed in relative positions so as to potentially use classified data in an unclassified manner; the important information is where the platforms are relative to one another, not where they are in an absolute sense. The locations are aggregated into cells of varying dimensions to enable more efficient calculations.

<sup>1</sup> See [www.gams.com](http://www.gams.com) for specific information on the solvers involved in GAMS.

<sup>2</sup> A mission is defined as a shooter/weapon pair firing upon a target.

The shooter laydown contains shooter names, quantities, and locations. The weapon laydown contains the same information in addition to the future value of the munition. The future value of the munition will limit the targets fired upon typically to those with higher values. Certain weapons can therefore be saved for opportunities to shoot high-value targets by raising the future value of those weapons. The target laydown contains the same information as the shooter laydown in addition to a value and a unit area for each target. The values of the targets are used to assess which missions to run, the overall value of the missions, and the total expected kills (discussed later).

Each type of shooter has associated with it a total number of volleys that it can fire. This is in addition to the total number of volleys a given shooter/weapon pair can fire. Each shooter is limited to certain weapons on the battlefield, and the weapons fired must be co-located with the shooter.

Munitions are classified as either direct or indirect. This determines whether collateral damage is assessed in the planning process. In the model, a direct fire weapon does not accrue any expected collateral damage, whereas an indirect fire weapon might.<sup>3</sup> The amount of expected collateral damage from indirect fire weapons is determined by the expected kills, range, and other factors described later in this chapter. Each munition has a cost associated with it. The cost is unitless in that the user can specify the units of measure to be calculated. For example, users interested in the monetary costs of munitions can use dollar values assigned to each munition to assess the total costs of a mission. Analysts interested in the weight of munitions can assign a mass value to each munition. The amount specified can be used as a constraint; for instance, if the user wants to limit the amount of munitions used in a given mission to a certain total cost, a cost constraint can be imposed to do so. This cost constraint is explained in the next section. Finally, munitions are only used with certain specified shooters and can only be used a certain number of times by any given shooter.

There are two types of targets. Positive targets are targets that a commander wants to destroy, for instance, enemy tanks and personnel carriers. Negative targets are targets the commander does not want to destroy, such as school buses and churches. These carry a negative value. There is a limit to how high the planned total expected kills can reach on any given target. The formulation is a planning device, and thus a user can plan to “overkill” a target by continually firing upon it with excess weapons. A weaponer may do this because the target has so high a value it is necessary to be certain that it is affected. Another reason may be that the weapon’s future value renders it useless relative to the value of the target. In this case, overkilling may be warranted. Either way, each target is assigned an “overkill” factor that limits the missions that can be fired against it.

The overkill value will typically be set to a value greater than 1 to ensure that the target is given the opportunity to be killed. For example, if a given shooter/weapon pair has an expected kill value of 0.6 against a target, it will not fire that mission if that target has an overkill value less than 0.6.

If a target is highly valued, it may be worthwhile to fire two missions of that shooter/weapon pair to get the total expected kill value closer to 1. If the overkill is set sufficiently high, say to 1.5, the second mission would be allowed because two missions with expected kills of 0.6 each will give a total expected kills of 1.2, which will be less than the limit

---

<sup>3</sup> The designation of “indirect” or “direct” in the model has only to do with the assumption of collateral damage for a given munition.

of 1.5 set on the target.<sup>4</sup> Thus, an overkill of more than 1 may be warranted in some circumstances.

### Constraints

Constraints in the formulation come in many types. The model can be used to produce a solution from two different angles. The first maximizes the total value of targets killed by all possible shooter/weapon pairs subject to specific constraints. The constraints will attenuate the extent of weapons used before all of the weapons are expended. The second solution is to set a desired value of targets killed and then minimize the budget to reach that value.

Shooters cannot fire a weapon more than a specified rate, and cannot shoot more than a total number of rounds in any one mission.

Each shooter/weapon pair must be in the same cell, and the maximum range that they can fire is set for each different type of target. For indirect weapons, the expected kills are not a function of range; rather, they are calculated from the total number of entities in the cell to which they are firing. For direct weapons, the expected number of kills when a shooter/weapon pair fires upon a target is range dependent. Although the purpose of the range constraint is to represent the ranges of the various shooters and munitions types, it may also be used to prohibit the use of certain types of munitions against targets in particular cells.

As noted previously, weapons have an associated cost. If a budget constraint is applied, the sum of all the weapons allocated cannot rise above that value. This value could be derived from the implications of doctrine, such as the controlled supply rates (CSR) and required supply rates (RSR). Similarly, the future value of a munition limits the quantity of weapons used—the value of the target must be higher than saving the munition for later use.

The internetting of fires concept is contained in many of these constraints. Specifically, the ability to adjust and explore shooter/munition pairing rules and information constraints will help to better understand the concept. With the current formulation, the trade-offs between different weapon stocks and shooter employments can be quantified.

### Origin of the Input Data

The tool was developed to accept data from many sources, from notional data used to prove a point or run speculative analyses, to real-life data to provide specific insights to commanders or military users. The analyst must assess what levels of precision and accuracy are needed in the inputs for any given analysis.

The data used in our example analysis (Chapter Three) were generated from a variety of military sources as shown in Table 1. The table shows that the bulk of the inputs were generated from data supplied from the Army Materiel Systems Analysis Activity (AMSAA). This data required considerable manipulation to get it into the correct format for the IOF Allocator. Force laydowns can be taken from a combat simulation and transformed into the needed locations. Expected kills for the different shooter/weapon/target combinations are

---

<sup>4</sup> This approximation on the total expected kills value assumes no systematic error and allows the formulation as a linear model.

**Table 1**  
**Sources for Some of the Data in the Study**

Type	Source	Comments
Force laydown	TRAC	Dynamic Scenario simulation runs
Expected kills for direct fire	AMSAA	Performance data → kill rates used in Vector in Commander (VIC) <sup>a</sup> → expected kills per round
Expected kills for indirect fire	AMSAA	Round reliability, angle of fall, aiming errors, precision errors in range and deflection → ART QUIK <sup>b</sup>
Expected kills for precision munitions	AMSAA	Round reliability * P(acquisition given round works) * P(kill given acquire) = P(kill) for each round
Threat laydown	TRADOC DCSINT	Threat support directorate
Shooter/munitions firing rate	AMSAA	FCS system book, Field Artillery Center, and military judgment based on operational rates
Costs, values, other	Military judgment	Trends are probably correct, but these will vary based on the commander

<sup>a</sup> VIC is an operational-level combat model developed by the Army.

<sup>b</sup> ART QUIK [7] is an artillery fire solution program developed by the U.S. Army Field Artillery School (USAFAS).

created from multiple sources and require manipulation to provide data in the correct format. There are several sources for the shooter/munition firing rates: either they are taken from the Future Combat Systems (FCS) handbook, Field Artillery Center, or military judgment is used to generate the data. Many of the inputs to the program are a result of military judgment. Future values of munitions and values of the targets are probably the most speculative, since this data is solely a result of military judgment and will change from commander to commander.

### Calculating Expected Kills

The calculations of expected kills as inputs to the model are discussed below. The method to compute the expected attrition from direct fire is based on probability of kill values. The expected attrition from indirect area fires is based on ARTQUIK 4.1 from AMSAA and USAFAS.<sup>5</sup> [7] The parameters and assumptions used are:

$VA_{s,m,t}$  The volley area for targets of type  $t$  shot at by shooters of type  $s$  with munitions of type  $m$

$UA_{s,m,t}$  The unit area for targets of type  $t$  (usually the cell area)

$FD_{s,m,t}$  The fractional damage to targets of type  $t$  shot at by shooters of type  $s$  with munitions of type  $m$

$N_{t,i}$  The number of targets of type  $t$  in cell  $i$

We assume that the targets are located uniformly throughout the unit area and that attrition to one target is independent of the attrition to others. Collateral damage cannot

<sup>5</sup>Area effects are range dependent; however, for simplicity in calculation, it was assumed that munitions are used at a fraction of their maximum range, and we used the expected fractional damage of a volley shot at that range in the calculation.

spread outside the boundaries of a given cell and thus, it is assumed that the volley is no larger than the size of the unit cell. The expected attrition of targets of type  $t$  from shooters of type  $s$  with munitions of type  $m$  is

$$FD_{s,m,t} \left[ N_{t,t} \cdot VA_{s,m,t} / UA_t \right].$$

Precision weapons are modeled as point-to-point direct fire weapons based on information from AMSAA as listed in Table 1.

### Limitations of the Model

Nonlethal fires and the effects of suppression were not considered. The mix of weapons and munitions was optimized for destructive fires only. Target location errors (TLE) and false identifications within the laydown are not explicitly represented, which may lead to an overestimation of weapon effectiveness. The effects of moderate TLE may be folded indirectly into the data if known. In addition, battle damage assessment (BDA) was assumed to be perfectly accurate, preventing the shooters from attacking dead targets (which also may lead to an overestimation of the damage to targeted systems). This too was not a limitation to the IOF Allocator but rather of the incoming data. Finally, the formulation is one-sided and does not explicitly model blue losses.

The Allocator is a planning device and, insofar as the data can be assumed to be the best possible at the time of the decision, a reasonable analysis may be to understand the affects of poor data on the solution. For instance, after the IOF Allocator has solved for the expected kills based on the assumed data, the Adjudicator can use the ground truth to assess the damages. The ground truth may be different from the information fed into the IOF Allocator formulation, and thus the next time around, the solution may show different outcomes than expected. Iterations along this line may afford an understanding of the value of information on weapon mix choices and combat outcome.

The expected-kills calculations assume that the targets are located uniformly throughout the cells. This is a means to aggregate the data somewhat to reduce the run-time of the analysis. Moving to a unit-based approach rather than a cell-based approach would be the next step in formulating the problem.

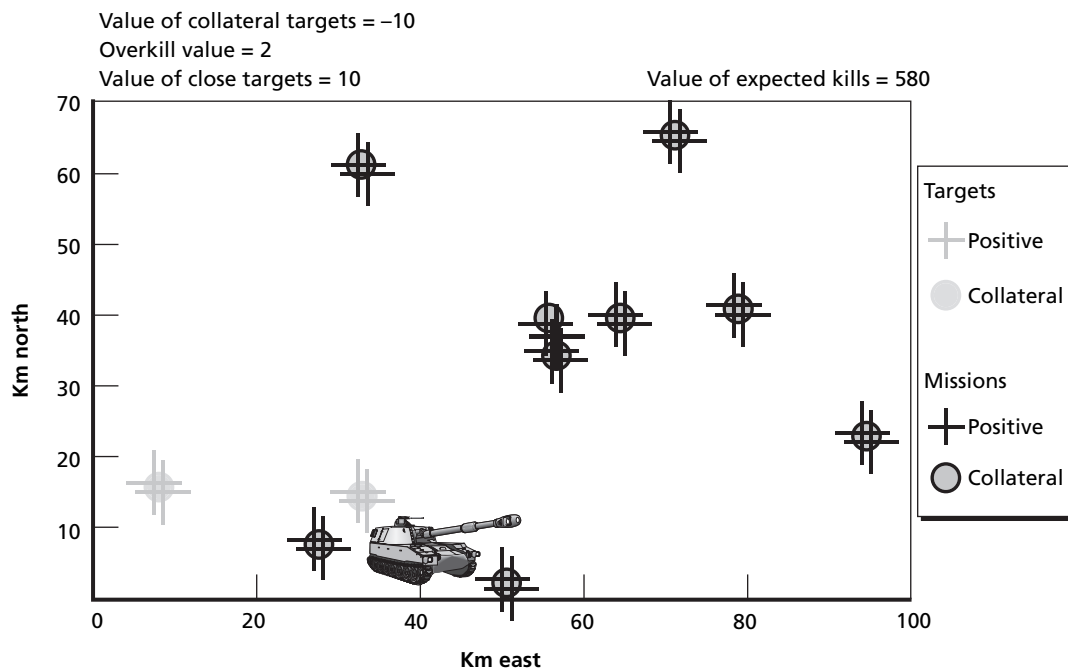
Time of flight of the munition is not considered in the analysis. A time-of-flight parameter associated with weapons may change the allocation if moving or fleeting targets are considered in the analysis.

## Example Analysis

In this chapter we discuss an example analysis using the IOF Allocator to highlight potential outputs and issues for discussion. We will initially show the effects of the collateral damage penalty on the solution, the effects of the overkill limit on the solution, and the effects of different target values on the solution. The example is for illustrative purposes only. The data used are unclassified and fabricated for proof of principle of the formulation. The figures were manually generated from the model outputs.

Figure 3 presents an example of targets on a 100km  $\times$  70km grid. There are several indirect fire shooters in grid cell (35, 3), represented by a self-propelled howitzer (SPH) icon. The figure shows many positive targets (tanks, shown as crosses) and collateral targets (shown as shaded circles) on the battlefield. In this example, there is at least one collateral target in every cell that there is a positive target. Tanks are assigned a value of 10, and collateral targets are assigned a value equal to the negative of one tank ( $-10$ ). There is an overkill factor of two in this example, meaning that the allocation is allowed to plan to kill twice the number of targets in a grid cell.

**Figure 3**  
**Allocation of Fires**



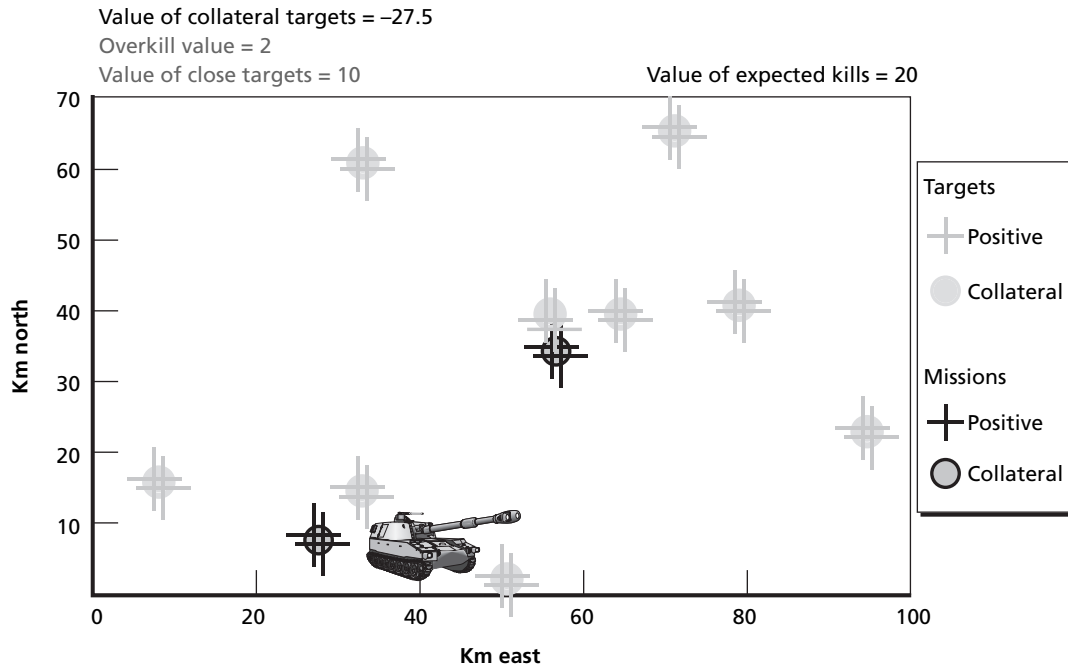


The IOF Allocator shows that many of the cells with targets are attacked, as indicated by the dark circles. The two areas with light gray targets (at (5, 18) and (32, 17)) were not fired upon. In this example, the total value of targets killed is 580. Perhaps not surprisingly, this is the value of adding both the positive targets and the negative targets (the latter reduces the total value expected).

Figures 4, 5, and 6 show the effects of changes to the initial scenario from:

- increasing the value of collateral targets,
- reducing the overkill value, and
- increasing the value of close enemy targets.

**Figure 4**  
**Allocation of Fires If the Penalty for Collateral Targets Is Increased**



RAND TR170-4

In Figure 4, the value of collateral targets across the battlefield was reduced from -10 to -27.5. This increased penalty for killing collateral targets resulted in a small number of fire missions in only the most lucrative grid cells. As shown in the figure, only two cells have missions planned. The threshold for this phenomenon was determined by examining the marginal values output from the IOF Allocator. The threshold was 25, at which point missions begin to be cut. If the penalty for collateral kills had been increased beyond 30, no attacks would have been allocated. In this figure, the total value of expected kills has dropped precipitously from 580 to 20 with the reduction in the number of missions that can be planned.

**Figure 5**  
**Allocation of Fires If the Overkill Factor Is Decreased**

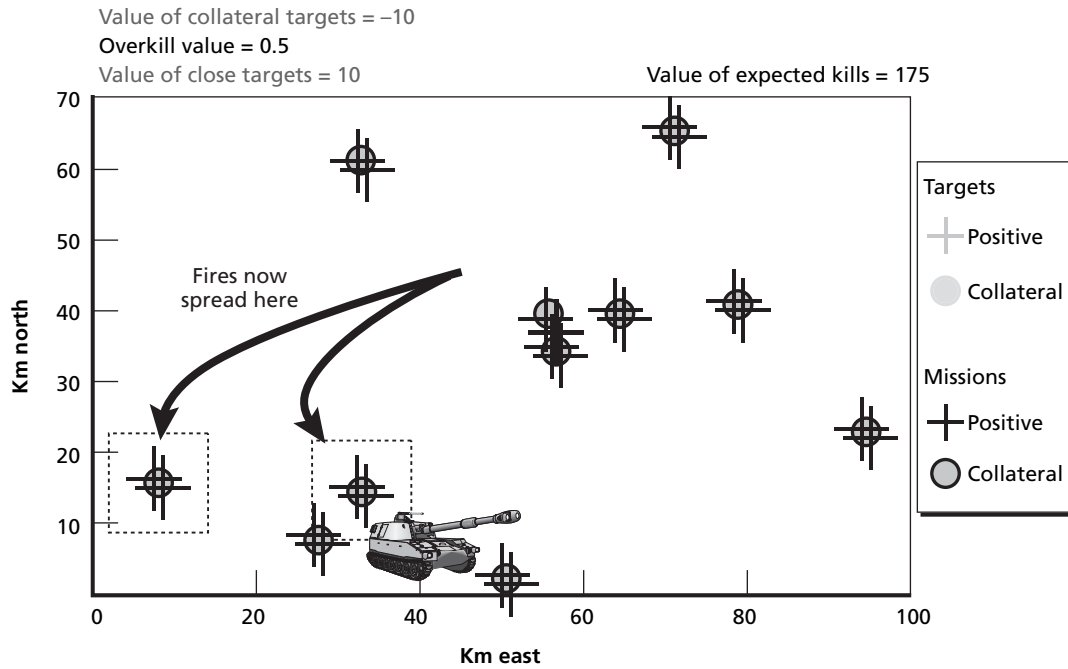
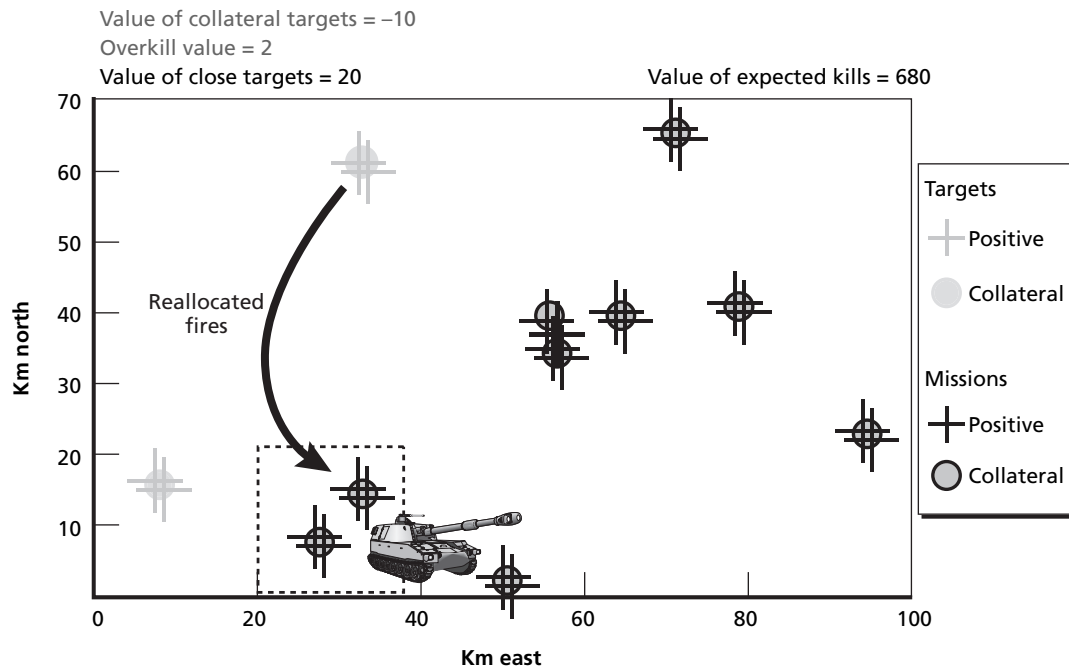


Figure 5 shows the allocation of fires if the overkill factor is reduced across the battlefield from 2 to 0.5. This means that at most, only one-half of the total number of a given target in a cell can be expected to be killed. With the mission reduction in some of the target-rich cells, fires are now allocated to cells that were previously not serviced (highlighted in the figure with gridded boxes). Subsequently, the reduced number of missions and spreading out of the missions to previously less valuable cells reduce the total value of expected kills from 580 to 175.

**Figure 6**  
**Allocation of Fires If the Value of the Close Targets Is Increased**



RAND TR170-6

If the value of the targets close to the shooter is increased from 10 to 20, fires are reallocated from less productive cells to those cells. Presumably the cells close to the shooter were not valuable enough to warrant as many missions to be planned. As shown in Figure 6, missions are allocated to a cell close to the SPH to take advantage of the higher-value targets there. Comparing Figure 3 and Figure 6, we see that the missions from the upper left were reallocated to the higher-value cells closer to the SPH. Subsequently, the total value of expected kills has increased from 580 to 680 as more valuable cells are targeted, which reflects the increase in the value of those targets.



## Insights and Future

---

This report has described the formulation of a tool for quantitatively assessing the effects of different weapons mixes on the expected value of targets killed. Furthermore, it provided a means of quantifying the benefit of internetting of fires through the ability to model the effects of different networks of shooters on target attrition.

In addition to the method we have described, the example analysis has produced some initial observations from the use of the IOF Allocator. The tool has allowed the formulation of follow-on questions that may be answered by a thorough analysis with the described method. The questions may help in gaining understanding of the type of conclusions that could be expected from a more complete analysis with the formulation described in this study. The questions can be summarized as follows:

- Is the selection of possible targets sensitive to available weapons?
- What is the effect of weapon accuracy on the choice of targets?
- What is the role in future forces for area munitions, and what is the appropriate mix of area versus point munitions?
- How does the contribution of various weapon systems change with changing enemy force composition and disposition?
- How dependent on range is the utilization of specific weapon systems?

### Future Improvements

The method described in this report has produced a tool to incorporate a number of the decisionmaking criteria for the allocation of fire missions. A number of potential improvements envisioned for the formulation are detailed below.

- **Unit-based approach.** The tool provides the ability for quick analysis by aggregating weapons, shooters, and targets geographically on the battlefield. An extension to the formulation would be to introduce a unit-based approach to the problem whereby the units are disaggregated on the battlefield. This will increase the number of calculations, but it will provide a more robust set of missions for the solution.
- **Nonlethal technologies.** As the formulation is not necessarily specific to lethal weapons, future analyses might include nonlethal technologies, for example, to provide counterpersonnel or countermaterial effects in the process. This may entail a better understanding of the effects of nonlethal weapons on targets, as well as the tradeoffs

between lethal and nonlethal weapons as they relate to the generation of the desired outcome.

- **Terrain.** The current formulation does not directly take into consideration terrain features when calculating expected kill rates. It could be useful to develop a terrain background that does influence the ability to engage targets.

As the Allocator becomes more versatile, incorporating more of the effects-generating capabilities available to commanders, it will offer the ability to inform weapons mix analyses and may provide some information on the allocation of assets between different echelons of the military.

The methodology employed may also be envisioned as a way of analyzing sensor mixes to fulfill the sensor allocation part of the Networked Fires triad; however, the probabilistic nature of sensor collection capabilities may make the application more difficult. These types of analyses could be used in war gaming efforts to help decisionmakers in near real time and for Future Force analysis.

## Mathematical Formulation of the Problem

---

In this appendix we formulate the static, snapshot problem as a mathematical program. We assume that the battlespace is partitioned into cells numbered  $i = 1, \dots, I$ . Let  $n_{t,i}$  be the number of targets of type  $t = 1, \dots, T$  in cell  $i$ , and let  $f_{s,i}$  be the number of shooters of type  $s = 1, \dots, S$  in cell  $i$ . If two independently operating weapon systems are mounted on the same platform, say a machine gun and a TOW launcher, this platform should be treated as two separate co-located shooters. We assume that collateral damage across cell boundaries is not possible. Collecting the targets and shooters into cells reduces the complexity of the formulation. We also assume that there are  $h_{m,i}$  munitions of type  $m = 1, \dots, M$  in cell  $i$  and that  $l_{s,m}$  equals one if shooters of type  $s$  may use munitions of type  $m$  and zero otherwise. The  $\{n_{t,i}\}, \{f_{s,i}\}, \{h_{m,i}\}, \{l_{s,m}\}$  arrays are direct data inputs. Our goal is to determine  $x_{s,m,i,t,j} \geq 0$ , the integer number of missions by shooters of type  $s$  with munitions of type  $m$  in cell  $i$  to be fired at targets of type  $t$  in cell  $j$ .

The usage of munitions is constrained by

$$x_{s,m,i,t,j} \cdot (1 - l_{s,m}) = 0, \forall s, m, i, t, j$$

and

$$\sum_{s=1}^S \sum_{t=1}^T \sum_{j=1}^I x_{s,m,i,t,j} \leq h_{m,i}, \forall m, i.$$

Let  $r_{s,m,i,t,j}$  equal one if shooters of type  $s$  in cell  $i$  may fire munitions of type  $m$  at targets of type  $t$  in cell  $j$  and zero otherwise. The range constraints may be stated as

$$x_{s,m,i,t,j} \cdot (1 - r_{s,m,i,t,j}) = 0, \forall s, m, i, t, j.$$

Although the purpose of this constraint is to represent the ranges of the various shooters and munitions types, it may also be used to prohibit the use of certain types of munitions against targets in particular cells. Let  $R_s$  be the maximum number of equivalent missions that shooters of type  $s$  may fire and  $e_{s,m}$  be the number of equivalent missions fired for each single mission of shooter  $s$  fired with munitions of type  $m$ . Typically,  $e_{s,m}$  equals one. The fire rate constraints are



$$\sum_{m=1}^M \sum_{t=1}^T \sum_{j=1}^I e_{s,m} \cdot x_{s,m,i,t,j} \leq f_{s,i} \cdot R_s, \forall s, i.$$

The  $\{r_{s,m,i,t,j}\}$ ,  $\{e_{s,m}\}$ ,  $\{R_s\}$  arrays are directly input.

The number of munitions of type  $m = 1, \dots, M$  used by shooters in cell  $i$  is equal to  $\sum_{s=1}^S \sum_{t=1}^T \sum_{j=1}^I x_{s,m,i,t,j}, \forall m, i$ . To include the cost of munitions into the problem we need  $c_m$ , the

cost of a single munition of type  $m$ . The total cost of munitions expended is then

$$\sum_{s=1}^S \sum_{m=1}^M \sum_{i=1}^I \sum_{t=1}^T \sum_{j=1}^I c_m \cdot x_{s,m,i,t,j}, \text{ which must satisfy}$$

$$\sum_{s=1}^S \sum_{m=1}^M \sum_{i=1}^I \sum_{t=1}^T \sum_{j=1}^I c_m \cdot x_{s,m,i,t,j} \leq B$$

for some budget figure,  $B$ .

Let  $p_{s,m,i,t,\tau,j}$  be a planning factor for the expected number of kills of targets of type  $\tau = 1, \dots, T$  in cell  $j$  by one mission of shooters of type  $s$  with munitions of type  $m$  in cell  $i$  fired at targets of type  $t$  in cell  $j$ . This formulation allows for collateral damage. The planning factor  $p_{s,m,i,t,\tau,j}$  is computed from the laydown of targets in the cell  $\{n_{\tau,i} \mid \tau = 1 \dots T\}$  and analyst judgment. Let  $\delta_\tau \geq 0$  be the global proportion of the targets of type  $\tau = 1, \dots, T$  that is allowed to be killed and  $d_{\tau,i}$  be the proportion of targets of type  $\tau = 1, \dots, T$  in cell  $i = 1, \dots, I$  that is allowed to be killed. Assuming that the results of fire missions are independent and that kills vary linearly with the number of missions,<sup>1</sup> the expected number of kills of targets of type  $\tau$  in cell  $j$  is

$$K_{\tau,j} = \sum_{s=1}^S \sum_{m=1}^M \sum_{i=1}^I \sum_{t=1}^T p_{s,m,i,t,\tau,j} \cdot x_{s,m,i,t,j}, \forall \tau, j.$$

A simple modification of this formulation will allow for a piecewise linear relationship between expected kills and missions. We restrict the number of expected kills with

$$K_{\tau,j} \leq \delta_\tau \cdot d_{\tau,j} \cdot n_{\tau,j}, \forall \tau, j.$$

Let  $v_{\tau,j}$  be the value of targets of type  $\tau$  in cell  $j$ . Our objective is to maximize the total value killed, namely,  $\sum_{\tau=1}^T \sum_{j=1}^I v_{\tau,j} \cdot K_{\tau,j}$ . Negative values of targets may be used to direct fire

<sup>1</sup> Necessary assumption to ensure a linear model.

missions away from cells where collateral damage might result in unintended targets, such as school buses, being killed. To facilitate this formulation, let

$$p_{s,m,i,t,\tau,j}^+ = \begin{cases} p_{s,m,i,t,\tau,j} & \text{if } v_{\tau,j} \geq 0 \\ 0 & \text{else} \end{cases}$$

and

$$p_{s,m,i,t,\tau,j}^- = \begin{cases} p_{s,m,i,t,\tau,j} & \text{if } v_{\tau,j} < 0 \\ 0 & \text{else} \end{cases}.$$

Note that  $p_{s,m,i,t,\tau,j} = p_{s,m,i,t,\tau,j}^+ + p_{s,m,i,t,\tau,j}^-, \forall s,m,i,t,\tau,j$ . Now we define

$$K_{\tau,j}^+ = \sum_{s=1}^S \sum_{m=1}^M \sum_{i=1}^I \sum_{t=1}^T p_{s,m,i,t,\tau,j}^+ \cdot x_{s,m,i,t,j}$$

and

$$K_{\tau,j}^- = \sum_{s=1}^S \sum_{m=1}^M \sum_{i=1}^I \sum_{t=1}^T p_{s,m,i,t,\tau,j}^- \cdot x_{s,m,i,t,j}.$$

Also note that  $K_{\tau,j} = K_{\tau,j}^+ + K_{\tau,j}^-, \forall \tau,j$ .

In order not to squander valuable resources, we will assume that the future value of munitions of type  $m$ ,  $V_m$ , is known, and we will require that

$$\sum_{\tau=1}^T v_{\tau,j} \cdot p_{s,m,i,t,\tau,j} \cdot x_{s,m,i,t,j} \geq V_m \cdot x_{s,m,i,t,j}, \forall s,m,i,t,j.$$

The value arrays,  $\{v_{t,j}\}$  and  $\{V_m\}$ , are the result of decisionmaker judgments. The value of the targets and the future value of the munitions are on the same relative scale.

The mathematical program for the value maximization problem then becomes

$$\max \sum_{\tau=1}^T \sum_{j=1}^I v_{\tau,j} \cdot (K_{\tau,j}^+ + K_{\tau,j}^-),$$

subject to

$$\sum_{s=1}^S \sum_{m=1}^M \sum_{i=1}^I \sum_{t=1}^T \sum_{j=1}^I c_m \cdot x_{s,m,i,t,j} \leq B,$$

$$\sum_{\tau=1}^T v_{\tau,j} \bullet \left( p_{s,m,i,t,\tau,j}^+ + p_{s,m,i,t,\tau,j}^- \right) \bullet x_{s,m,i,t,j} \geq V_m \bullet x_{s,m,i,t,j}, \forall s,m,i,t,j,$$

$$K_{\tau,j}^+ = \sum_{s=1}^S \sum_{m=1}^M \sum_{i=1}^I \sum_{t=1}^T p_{s,m,i,t,\tau,j}^+ \bullet x_{s,m,i,t,j}, \forall \tau,j,$$

$$K_{\tau,j}^+ \leq \delta_{\tau} \bullet d_{\tau,j} \bullet n_{\tau,j}, \forall \tau,j,$$

$$K_{\tau,j}^- = \sum_{s=1}^S \sum_{m=1}^M \sum_{i=1}^I \sum_{t=1}^T p_{s,m,i,t,\tau,j}^- \bullet x_{s,m,i,t,j}, \forall \tau,j,$$

$$K_{\tau,j}^- \leq \delta_{\tau} \bullet d_{\tau,j} \bullet n_{\tau,j}, \forall \tau,j,$$

$$\sum_{m=1}^M \sum_{t=1}^T \sum_{j=1}^I e_{s,m} \bullet x_{s,m,i,t,j} \leq f_{s,i} \bullet R_s, \forall s,i,$$

$$\sum_{s=1}^S \sum_{t=1}^T \sum_{j=1}^I x_{s,m,i,t,j} \leq h_{m,i}, \forall m,i,$$

$$x_{s,m,i,t,j} \bullet \left( 1 - r_{s,m,i,t,j} \right) = 0, \forall s,m,i,t,j,$$

$$x_{s,m,i,t,j} \bullet \left( 1 - l_{s,m} \right) = 0, \forall s,m,i,t,j,$$

with nonnegativity and integrality constraints

$$\text{Integer } x_{s,m,i,t,j} \geq 0, \forall s,m,i,t,j,$$

$$K_{\tau,j}^+ \& K_{\tau,j}^- \geq 0, \forall \tau,j.$$

To reformulate this as a cost-minimization problem, the objective function becomes  $\min \sum_{s=1}^S \sum_{m=1}^M \sum_{i=1}^I \sum_{t=1}^T \sum_{j=1}^I c_m \bullet x_{s,m,i,t,j}$ , the budget constraint is removed, and another constraint must be added in its place to require a certain amount of value to be killed, namely,

$$\sum_{\tau=1}^T \sum_{j=1}^I v_{\tau,j} \bullet \left( K_{\tau,j}^+ + K_{\tau,j}^- \right) \geq \Lambda$$

for some  $\Lambda$ .



## Run-Time Analysis

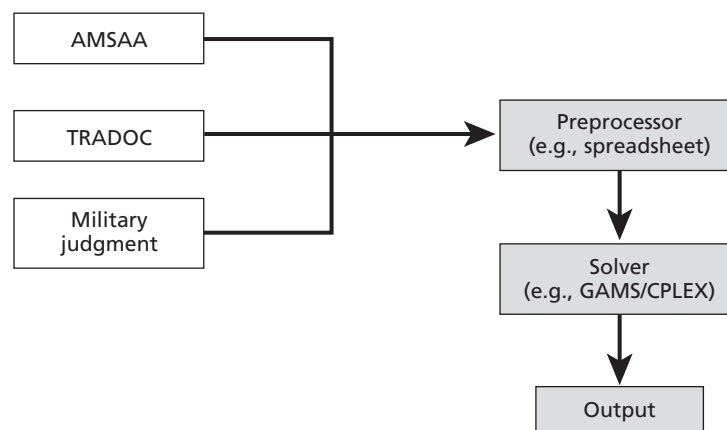
---

As with any model or analytical tool, it is necessary to know what the limitations of the IOF Allocator are with regard to run-time<sup>1</sup> and complexity of use. This appendix describes the steps involved in running the IOF Allocator and the approximate time to complete each step as a function of data.

The most time-consuming aspect of using the IOF Allocator effectively is in the generation of input data. The model was originally crafted to receive simple data and transform it into the specific inputs required by the math program solver. This would facilitate the use of data that might be readily available or easily understood by the analysts wanting to use the model. Therefore, it is helpful to better explain the specific characteristics of the inputs, the transformations that take place within the preprocessing, and the time it takes to generate the solution from the solver.

The model is run in three steps, as shown in Figure B.1. The first step is to generate data necessary to fill a preprocessing program such as a spreadsheet. The data in the spreadsheet contains: the laydowns of the shooters, munitions, and targets; information about each of the shooters, munitions, and targets such as overkill factors, costs, and firing rates; and range limitations and expected kill values for different shooter/munition/target combinations.

**Figure B.1**  
**Flow Chart of Model Process**



RAND TR170-7

---

<sup>1</sup> Here, run-time is taken to mean the approximate time necessary to set up the model, run it, and collect the outputs.

The second step is to generate the input for the solver from the data in the spreadsheet. A math program solver such as GAMS/CPLEX can take data in a variety of forms, and thus the spreadsheet needs to consistently generate the correct inputs to the solver.

The third step is to run the optimizer and collect the output.

The IOF Allocator can run on a desktop PC that has a spreadsheet program and a mathematical solver installed. The approximate times for each of the steps, depending on the precision of the data and the size of the scenario, are shown in Table B.1.

Using a spreadsheet offers the ability to make changes to the scenario quickly and easily, at the cost of having to run the preprocessor again to create the input to the math solver. The ability to make quick changes to the scenario also enables exploratory analyses and quick-reaction analyses.

**Table B.1**  
**Run-time Analysis of the Different Steps Involved in the Methodology**

Step	Precision or Size of Scenario <sup>a</sup>	Time to Completion
Finding and/or manipulating data input for the spreadsheet	Notional	< 1 hour the first time
	Military certified	Hours to days for the first time
Generating the input to the solver	Small	< 10 seconds
	Large	< 60 seconds
Solving program and generating output	Small	< 5 seconds
	Large	< 10 seconds

<sup>a</sup>A small scenario may contain 150 platforms spread across shooters and targets; a large scenario may contain up to 1,000 total platforms.

## References

---

1. Bauman, Mike, *Request for Operations Research: Internetting of Fires*, TRAC Memo, 14 February 2002.
2. Alberts, David, John J. Garstka, and Frederick P. Stein, *Network Centric Warfare: Developing and Leveraging Information Superiority*, Department of Defense C4ISR Cooperative Research Program (CCRP) Publication Series, 1999.
3. Alberts, David, John J. Garstka, Richard E. Hayes, and David A. Signori, *Understanding Information Age Warfare*, Department of Defense C4ISR Cooperative Research Program (CCRP) Publication Series, 2001.
4. Gonzales, Dan, and Louis R. Moore, et al., *Assessing the Value of Information Superiority for Ground Forces—Proof of Concept*, Santa Monica, CA, RAND, DB-339-OSD, 2001.
5. U.S. Army TRADOC, *Tactical Operational and Organizational Concept for Maneuver Units of Action*, TRADOC Pamphlet 525-3-91, Fort Monroe, Virginia.
6. Brooke, Anthony, et al., *GAMS, A User's Guide*, GAMS Development Corporation, December 1998.
7. JTCG/ME Surface-to-Surface Working Group, *ARTQUIK Computer Program User Manual*, Draft, May 14, 2002.
8. Objective Force Task Force, Networked Fires working definition and concept (26 Aug 02).
9. U.S. Army TRADOC, *Objective Force Operational and Organizational Plan for Maneuver Unit of Action*, TRADOC Pamphlet 525-3-90/O&O, Fort Monroe, Virginia, 22 July 2002.
10. U.S. Joint Chiefs of Staff, *Command and Control for Joint Air Operations*, Joint Pub 3-56.1, Washington, D.C., 14 November 1994.
11. Libicki, Martin, *Illuminating Tomorrow's War*, McNair Paper 61, Institute for National Strategic Studies, Washington, D.C., November 1999.
12. For examples, see Hura, Myron, et al., *Enhancing Dynamic Command and Control of Air Operations Against Time Critical Targets*, Santa Monica, CA, RAND, 2002.
13. TRADOC Analysis Center, "Objective Force C4ISR Modeling and Analysis Work Group (MAWG) Information Briefing," 25 September 2002.
14. Olwell, David, and Washburn, Alan, *Internetting of Fires*, Monterey, CA: Naval Postgraduate School, NPS-OR-02-003-PR, June 2002.
15. Havens, Michael E., *Dynamic Allocation of Fires and Sensors*, Monterey, CA: Naval Postgraduate School, Thesis, September 2002.



16. Endsley, M. R., Bolte, B., and Jones, D. G., *Designing for Situation Awareness—An Approach to User-Centered Design*, New York: Taylor and Francis, Inc., 2003.
17. Ackoff, R. L. “From Data to Wisdom,” *Journal of Applied Systems Analysis*, Vol. 16, 1989, pp. 3–9.